THE DEPENDENCE OF INFORMATION ENTROPY ON THE MEASUREMENT ERROR OF THE SENSOR NETWORK DESIGNATED TO THE TARGET LOCALIZATION

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Abstract: The Information from the sensors and resources which are available to the commander are important during his decision-making process. An important parameter of this information is their quality. This article describes the approach to processed signals using information entropy as a parameter for evaluating the quality of information. Initial precondition for applying the information entropy model, the individual optimization strategies are used in 2D and 3D space. In the previous work, we dealt with sensors which were capable to scan without measuring error. In this publication, we are dealing with the simulation of sensors dislocation which is affected by the same measurement error or different error for each individual sensor. Lastly, the thesis for possible future studies can be found at the end of this article.

Keywords: NEC, Network enabled capability, UAV, sensors, TDOA, information entropy.

1 INTRODUCTION

In the information age, in addition to the standard equipment which armies require for their combat, additional information and analytical tools are necessary for to increase the level of command and control process effectivity. In particular, this concerns the systems capable of combat situations recognition, sensors and tools for sensors information analyses.

The principal of the TDOA method was described in the paper [1]. This method is vastly used while testing and evaluating the effectivity of the estimation process. The paper [2] was focused on factors which influence sensors dislocation in 2D space and the optimization of their dislocation for the most accurate estimate of target's position. In the article [3] we applied a mathematical model in 3D space and, similarly to the previous article, we were dealing with the optimization accordingly.

The following article describes the relationship between the information and its impact on the battle results in the network-oriented NEC environment. In addition, this model applied in other procedures which describe the relative domination of information as well. While acquiring information from sensors and resources, the commander requires and selects the information which is the most valuable for his decision to be made. Therefore, the processing of signals from sensors and other sources will have substantial impact on the commander's decision. In a network-oriented environment, for to accomplish the information dominance, it is essential to achieve a certain degree of sensors and resources deployment effectivity as a considerable ratio to the quality and the information integrity.

2 MATHEMATICAL MODEL

We assume that our sensors are carried by UAV, they are able to communicate within the network and are capable of moving in the trajectory represented by perimeter of the sphere. Secondly, we assume that all sensors should have the different error deflection and are able to move in the same speed. The target followed by sensors is situated in the middle of the sphere.

During the testing and evaluation of effectiveness of estimate, the Cramer-Rao inequality is used. The main goal is to express a lower bound on the variance of estimators.

Cramer-Rao inequality (CRB) for target vector $\bar{p} \in R^D$ and sensors $\bar{q_i} \in R^D$, where D expresses 2 or 3 dimensional area and M expresses the quantity of sensors, can be defined by [4]:

$$CRB = J^{-1} = (v\sigma)^2 (GG^T)^{-1} \qquad (1)$$

where:

$G = [g_{ij\dots}]$, (i,j)∈ I, $\bar{g}_{ij} = \bar{g}_i - \bar{g}_j$, $\bar{g}_i = \frac{\bar{q}_i - \bar{p}}{\|\bar{q}_i - \bar{p}\|}$

where:

J ... is the Fischer information matrix (FIM), (its presence ensure the existence of linear independence of vectors),

$\bar{g}_i$ ... is the vector heading from the target p to sensor i,

$\bar{g}_{ij}$ ... is difference between two direction vectors,

σ^2 ... expresses an error variance caused by Gauss noise (reliability factor).

Set I consists of each individual sensor pair (i,j). Matrix G contains all vectors $\bar{g}_{ij}$, where (i,j)∈ I.

The most common strategy is to minimize the trace of CRB [4]:

$$min\, f_{CRB} = tr[J^{-1}] = (v\sigma)^2 tr[(GG^T)^{-1}]\,. \qquad (2)$$

Required conditions for calculation min f_{CRB} are:

1. $\sum_{i=1}^{M} \bar{g}_i = \bar{0}$
2. For matrix $D\,x\,M$ $g = [g_1 \dots g_M]$ must be $gg^T = \frac{M}{D} I$

where:

$\bar{g}_i$ ... is a vector heading from target p to sensor i,

M ... the number of sensors,

D ... area dimension,

I ... expresses matrix, where elements on the main diagonal of the matrix are equal to 1.

If we assume that sensors are deployed equally around the sphere perimeter, then the formula (2) can be replaced as follows [5]:

$$\sum_{i=1}^{M} c_i^2 g_i\, g_i^T = \frac{1}{D}\sum_{i=1}^{M} c_i^2\, I_D \quad (3)$$

where:
c_i ... is error variance of the sensor *i,*
g_i ... is vector pointing from the target to the sensor *i,*
M ... is the number of sensors,
D ... is dimension,
I_D ... is matrix of which diagonal values are equal to 1.

If previous equality is valid, then we can assume that the sensors are dislocated optimally and the minimum value can be expressed as following:

$$\Delta = \sum_{i=1}^{M} c_i^2 g_i\, g_i^T - \frac{1}{D}\sum_{i=1}^{M} c_i^2\, I_D.$$

3 RELIABILITY FACTOR AS A PARAMETER FOR EVALUATING THE QUALITY OF INFORMATION

In general, parameter c_i represents a reliability factor, or basically, a sensor measurement error. In essence, it represents the sum of probabilities of the sensor which affect its final deviation and thus the whole process of detection and optimal deployment.

The initial probability distribution and therefore the total entropy depends on the specific information available to the commander from each individual sensor. In each step of the optimization process of the sensor matrix dislocation, the probability distribution and overall information entropy evolves.

The probability distribution can be affected by the following factors:

- The amount of both, confirmed and unconfirmed messages from sensors;
- The reliability of sensors;
- Field conditions;
- Time latency between the information gathered;
- Vibration of sensors, etc.

We can express this measurement error in more granularly as following:

$$c_i = \frac{1}{(v\sigma)^2} \quad (4)$$

where:
v^2 ... expresses an error variance caused by vibrations of sensors,
σ^2 ... expresses an error variance caused by Gauss noise (reliability factor).

We can assume, that we have *i* sensors in operation area and A_1, $A_2,\ldots,A_n$ are independent events, which are considered as a target detections from each sensor and also that all sensors would follow just one target. For the final probability of the target detection stands:

$$P(\cup_{n=1}^{n} A_i) = 1 - P\left(\overline{\cup_{n=1}^{n} A_i}\right) = 1 - P\left(\overline{A_1}\right).P\left(\overline{A_2}\right)\ldots P\left(\overline{A_n}\right) \quad (5)$$

The reliability factor can be observed from different angles. In essence, the reliability factor represents the reliability of the sensors and the reliability of the information processing. In general, the reliability factor describes the quality of the information and the quality of the information network that allows the transfer of information from the sensors to the commander. In a network where each individual sensor is separated from the information network, the reliability factor expresses the entire quality of the sensor. Essentially, if required, this approach can applied across each individual NEC level.

4 THE DEPENDENCE OF INFORMATION ENTROPY ON THE SENSOR NETWORK MEASUREMENT ERROR

Once the information from all sensors is processed, the commander's level of knowledge changes with each iteration which is reflected in changes in probability distribution.

The current amount of information can be measured by information entropy. Information entropy is the measurement of the average amount of information in the probability distribution. The entropy function achieves the maximum when the value of the information in the probability distribution is the lowest (with the highest uncertainty). In practice, this happens if the commander nor possess the localization equipment, neither dispose prior information about the targets in the battlefield.

In this case, we can express the maximum information entropy as following:

$$H(U) = -\sum_{i=0}^{n} \frac{1}{n+1} ln \frac{1}{n+1} = ln(n+1) \quad (6)$$

The entropy function has its minimum value at zero. This phenomenon occurs when P(U) = 1 and represents a state of maximum certainty or minimal uncertainty. In our case, the entropy function has a minimum if the sensor matrix is in the optimal position.

To be able to simulate the dependence of the resulting information entropy on the individual sensor error, we had to ensure that the initial sensor configuration was not randomly generated, but that it was constant for all measurements.

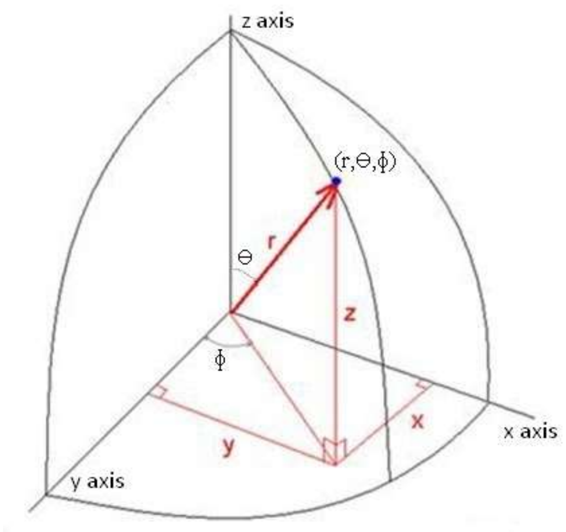

Fig. 1 Converts from Spherical to Cartesian coordinates in 3-dimensions

We know from geometry that there are five different solids whose vertices are located symmetrically around the surface of the sphere. In our case - for simulation purposes, we used the solid called the tetrahedron, which has four vertices.

At the beginning of the simulation, four sensors were randomly distributed over the surface of the sphere and are interpreted by blue points.

The coordinates of each individual sensor g_i are expressed by $[x, y, z]^T$(see Fig.1), where:

$$x = r.cos\phi.sin\theta$$
$$y = r.sin\phi.sin\theta$$
$$z = r.cos\theta$$

In the first iteration, the uniform array of optimal sensor dislocations, which is shown in green in the figure, was rotated around the center in order to identify the minimum time value necessary for sensors re-dislocation from initial location to the optimal.

In the second step, based on the formula (3) and considering criteria reflecting the minimum necessary time for initial configuration changes to the optimal setting, the sensors were re-dislocated to the optimal locations.

Figure 2 shows a simulation in which four sensors are placed and their measurement was not affected by the measurement error. Information entropy in this case reaches a zero value because the sensor matrix at the optimal position does not generate a measurement error.

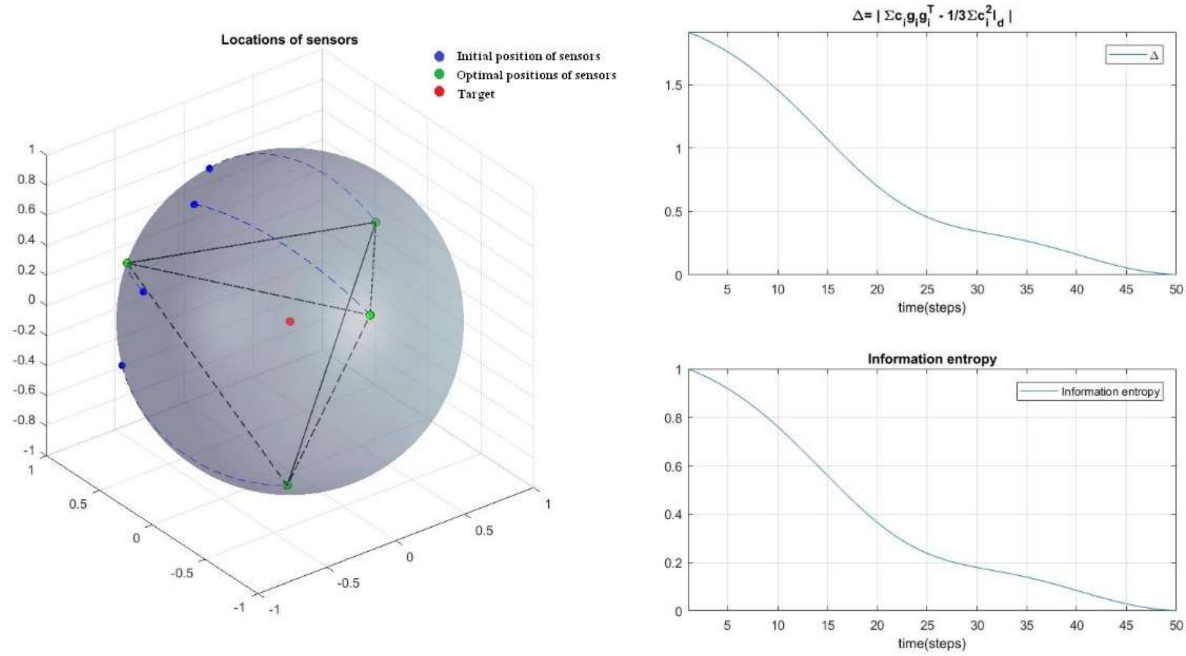

Fig. 2 Dependency of information entropy from measurement error (sensors are not affected by measurement error)

Figures 3 and 4 show the situation where all four sensors are affected by the same measurement error. The error of measurement for the case in Figure 3 was randomly set to 1.05 for all sensors, and for the case in Figure 4, the error was randomly set to 1.20. These values were set to correspond to approximately realistic conditions in practice. The difference between the individual simulations is only in the difference Δ between the actual value $\sum_{i=1}^{M} c_i^2 g_i\, g_i^T$ and its minimum value $\frac{1}{D}\sum_{i=1}^{M} c_i^2\, I_D$ is in each step relatively larger or smaller depending on the level of the measurement error and its orientation. The shape of the continuous variance curve between the optimal and real position of the sensors remained unchanged and therefore the same as in Figure 2. From the graph of the variance between the predicted position and the actual one we derived a graph of information entropy that is directly proportional to the variance. Information entropy in this case does not reach a zero value because the matrix sensor in the optimal position still generates a certain measurement error.

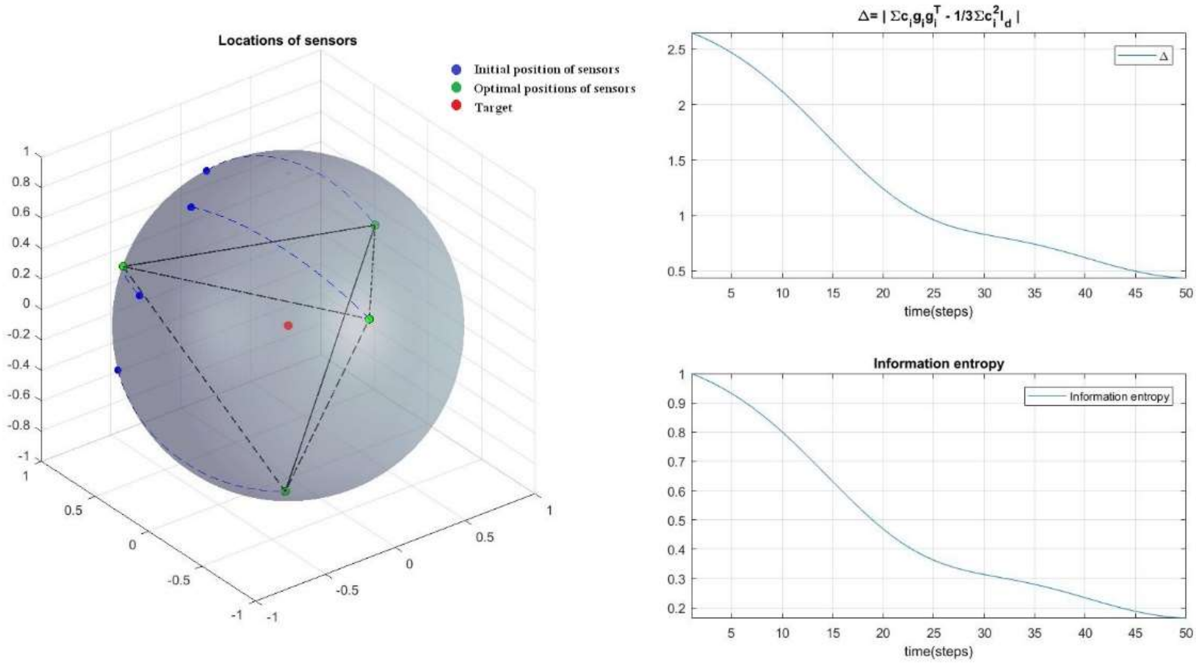

Fig. 3 Dependency of information entropy from measurement error (measurement error for all sensors was 1.05)

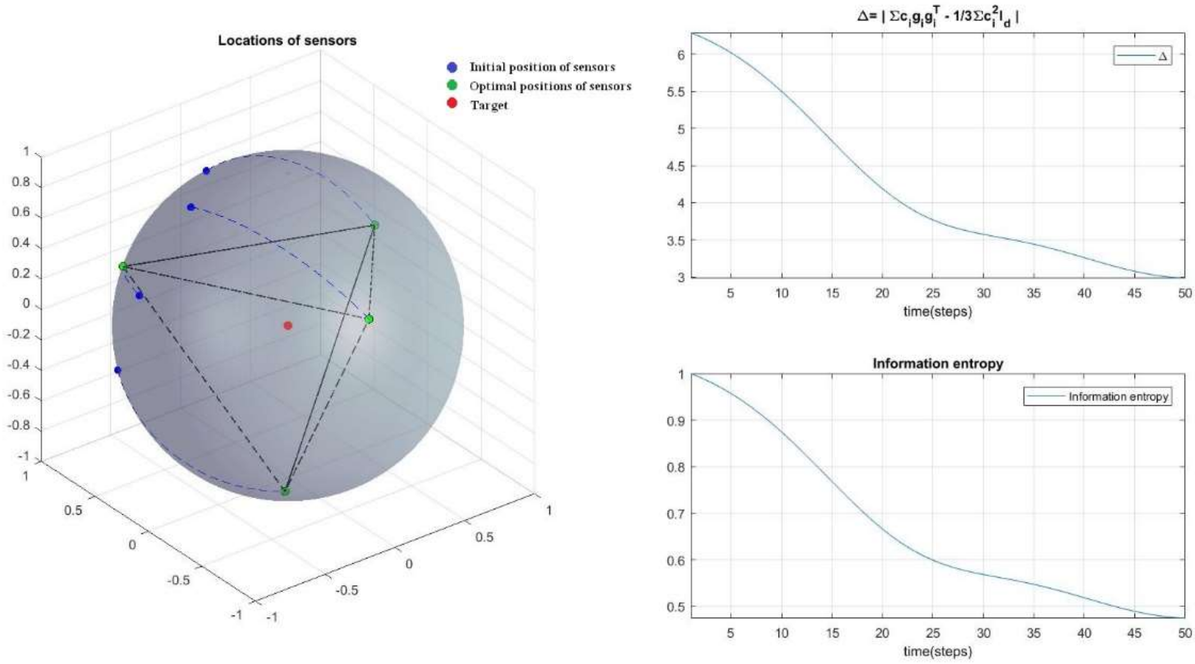

Fig. 4 Dependency of information entropy from measurement error (measurement error for all sensors was 1.20)

Figures 5 and 6 show a situation where each of the four sensors has a different measurement error. The measurement error for the case in Figure 5 was randomly set to c=[1 2 3 4] for each sensor, and for the case in Figure 6 the error was randomly set to c=[1 3 6 10]. These values are in practice not realistic and have been set as to clearly see the change in the shape of the variance graph and as well as the graph of information entropy. The shape of the continuous variance curve between the optimal and the real position of the sensors is different from the previous simulations and is dependent on the variance of each individual sensor. From the graph of the variance between the predicted position and the actual one we derived a graph of information entropy that is directly proportional to the variance. Information entropy in this case does not reach a zero value because the matrix sensor in the optimal position still generates a certain measurement error.

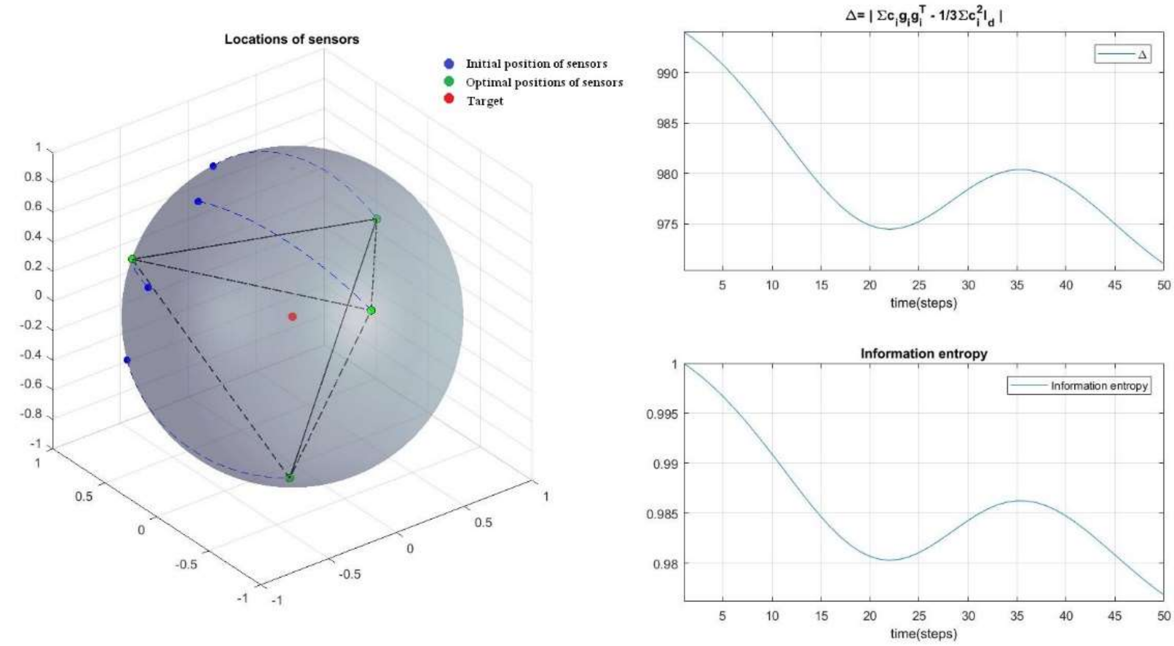

Fig. 5 Dependency of information entropy from measurement error (measurement error for all sensors was set to c=[1 2 3 4])

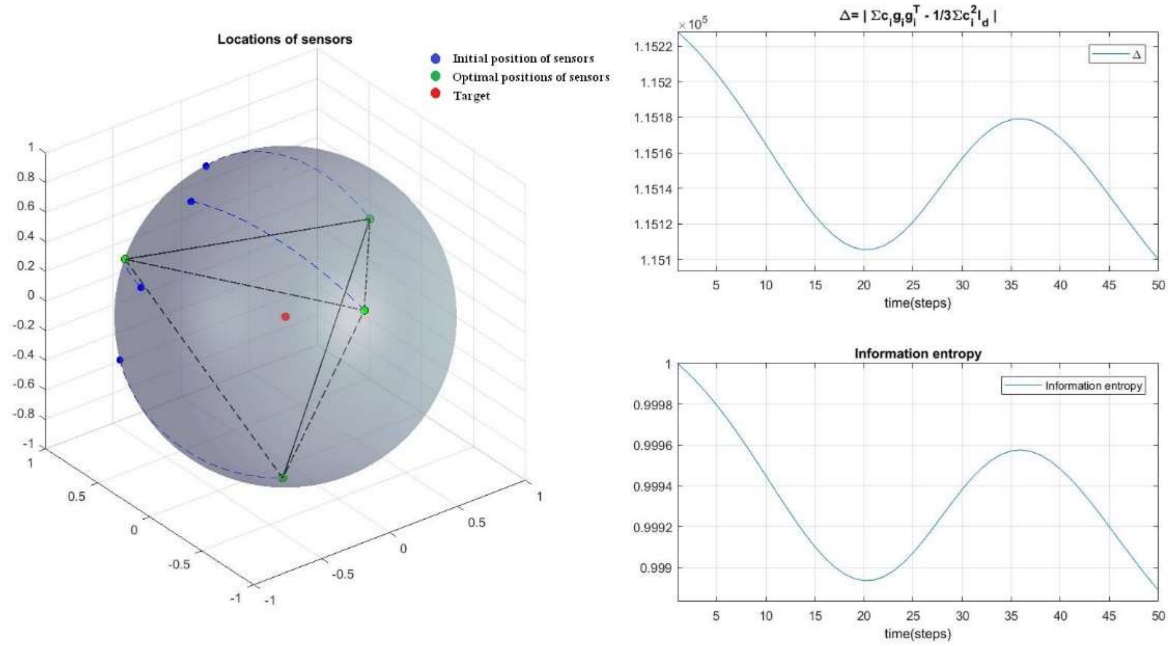

Fig. 6 Dependency of information entropy from measurement error (measurement error for all sensors was set to c=[1 3 6 10])

5 CONCLUSION

During the combat operation, the reconnaissance is as important as the effective use of firepower. The quality of the information depends on its accuracy, completeness and timeliness. The Common Operational Picture (COP) is an awareness of the battlefield situation and therefore represents the understanding of the opponent's intentions and the acceptance of effective actions and the subsequent decision of the commander. During the acquisition of information from sensors and other sources, the commander requests and selects information that has informative value for him. We have described the situation on the battlefield using a mathematical model and a probability model. From the graph of the variance between the predicted position and the actual one we derived a graph of information entropy that is directly proportional to the variance. Based on the change in each individual sensor measurement error, we showed a change in the variance graph between the predicted and the actual position and the corresponding information entropy. If we can predict the number of sensors available to the commander and we know on the basis of sensor properties (probability of detection), field conditions (location of sensors) and experience (weather conditions, sensor vibration) the approximate variance values for individual sensors, we can give the commander a more accurate view of the quality of the information obtained from this group of sensors. This model can also be useful in quantification of information superiority and in creating an optimized model of information superiority, which will be the subject of further modeling and exploration.

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